TAPERED WAVEGUIDE TRANSITIONS BETWEEN ARBITRARY CROSS SECTIONS AND SIZES

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Abstract

To analyze waveguide transitions between arbitrary cross sections and sizes the variational method of Galerkin is suitable for calculating local normal modes in intermediate cross sections. Near cutoff sections of spurious modes the solution of the generalized telegraphist's equations in form of Airy functions may be matched to the normal solution away from cutoff. By way of example spurious mode generation is analyzed for various square to round waveguide transitions.

Introduction

Tapered transitions between waveguides of different cross sectional shapes and sizes very often require intermediate cross sections of complex shape. The dominant mode transition from rectangular to round waveguide is a typical and important example, where waveguides with well known sets of eigenfunctions are joined by intermediate cross sections the eigenfunctions of which are not readily available. In addition tapered waveguide transitions are of interest where one or even both waveguides to be joined are of complex shape with no known set of modes. The respective application of the particular waveguide transition may require minimum return loss in as short a length as possible. Also one or both end-cross sections may be multimode and it may then be required that power conversion between the desired mode and any of the propagating unwanted modes shall be minimal again for a short length.

Analysis

To analyze such tapered waveguide transitions and design them optimally with respect to the particular requirements generalized telegraphist's equations are used and the field is represented by the local normal modes of the intermediate cross section.

To calculate local normal modes for subsequent implementation in the generalized telegraphist's equations the variational method of Galerkin [1] was found to be suitable. It adapts to any cross section with an inner point from which the radius vector to the wall is a single valued function of the azimuthal angle. The mode functions are given in terms of sines and cosines of boundary functions.

To test this method and the subsequent taper analysis a square to round waveguide transition with rounded squares as intermediate cross section was chosen as an example of practical significance. FIG.1 and 2 show the cutoff wavenumbers of the E- and H-modes in the intermediate cross sections.

For the nonuniform rectangular waveguide with rounded corners a representation of the generalized telegraphist's equations in terms of traveling waves leads to coupling coefficients between waves i and k which may be written as

$$K^{+}_{ik} = \frac{1}{2}\left(-\,C_{ik}\sqrt{\frac{Z_k}{Z_i}} + C_{ki}\sqrt{\frac{Z_i}{Z_k}}\right)$$

for coupling between waves traveling in the same direction, and

$$K^{-}_{ik} = \frac{1}{2}\left(C_{ik}\sqrt{\frac{Z_k}{Z_i}} + C_{ki}\sqrt{\frac{Z_i}{Z_k}}\right)$$

for coupling between waves traveling in opposite directions. Z_i designates the wave impedance of the mode i. The factor C_{ik} consists of three terms

$$C_{ik} = C^{I}_{ik}\,\frac{1}{D}\frac{dD}{dz} + C^{II}_{ik}\,\frac{dr}{dz} + C^{III}_{ik}\,\frac{dd}{dz}$$

where z is the axial coordinate and d the ratio of the narrow to the wide side of the rectangle. FIG. 3, 4 and 5 show the three factors C^{N}_{ik} and together with

$$C^{I}_{(1)(1)} = C^{I}_{(2)(2)} = -\,1$$

and

$$C^{I}_{[1][3]} = C^{I}_{(3)[1]} = C^{I}_{(4)[1]} = 0$$

and with FIG.1 and 2 provide all the necessary information for the respective taper analysis and design.

According to FIG. 3, 4 and 5 the dominant $(H^{\square}_{1o} - H^{\circ}_{11})$-mode is most strongly coupled to the $(E^{\square}_{12} - E^{\circ}_{11})$ and to the $(E^{\square}_{14} - E^{\circ}_{12})$-mode. Because of the smaller difference in the cutoff frequencies mode interaction between the dominant mode and the $(E^{\square}_{12} - E^{\circ}_{11})$-mode is most critical and it only needs to be considered.

In solving the generalized telegraphist's equations particular difficulties are encountered when the most critically coupled spurious mode goes through cutoff within the taper [2]. In this case the coupled differential equations are near the cutoff cross section transformed to current and voltage variables and the variable coefficients are expanded into power series thereby obtaining an Airy-differential equation with Airy-functions as solutions. Upon matching this solution near the cutoff cross section to the normal solution away from cutoff spurious mode generation may be calculated.

For the test example of the square to round waveguide taper with its intermediate cross sectional dimensions written as

$$D(z) = D_s + (D_r - D_s)\, g(z)$$

$$r(z) = \frac{1}{2}\, g(z)\, D(z)$$

where D_s is the square waveguide width and D_r the round waveguide diameter. The following transition functions were analyzed:

(1) $$g(z) = \sin^2 \frac{\pi z}{2L}$$

(2) $$g(z) = \frac{1}{30}\left(19 - 14 \cos \frac{\pi z}{L} - 4 \cos \frac{2\pi z}{L} - \cos \frac{3\pi z}{L}\right)$$

(3) $$g(z) = 0{,}6 - \frac{25}{48} \cos \frac{\pi z}{L} - \frac{4}{30} \cos \frac{2\pi z}{L} + \frac{1}{30} \cos \frac{4\pi z}{L} + \frac{1}{48} \cos \frac{5\pi z}{L}$$

(4) $$g(z) = \frac{1}{1 + (2 - \frac{2z}{L})^7}$$

with L the length of the taper.

Conclusion

In FIG.6 the spurious mode generation in a square to round waveguide transition for the different transition functions shows that much can be gained by choosing a suitable taper form.

References

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2. B.Z.Katzenelenbaum, Long symmetrical waveguide junction for H_{01}-wave. Radiotechnika i elektronika 2 No.5, 1957, p.531-546.

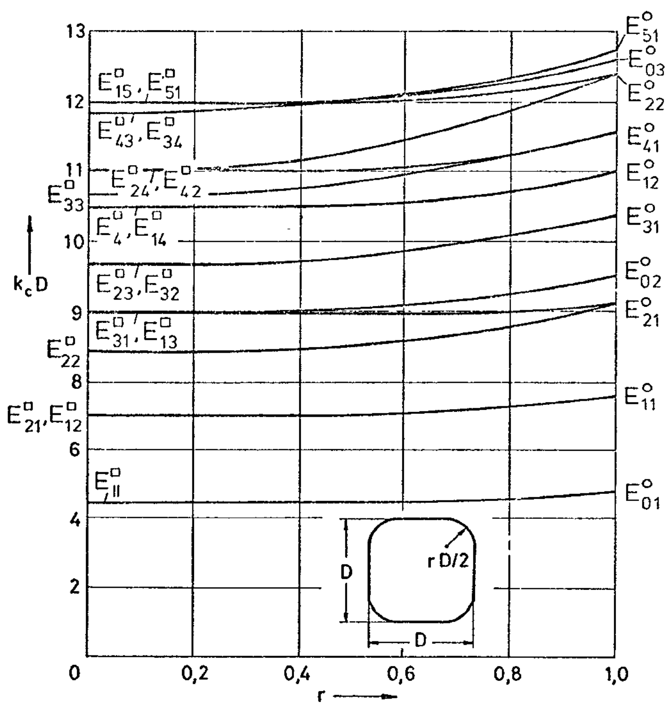

FIG.1 CUTOFF WAVE NUMBER k_c OF E-MODES IN SQUARE TO ROUND WAVEGUIDE TRANSITIONS WITH CORNERS ROUNDED OFF

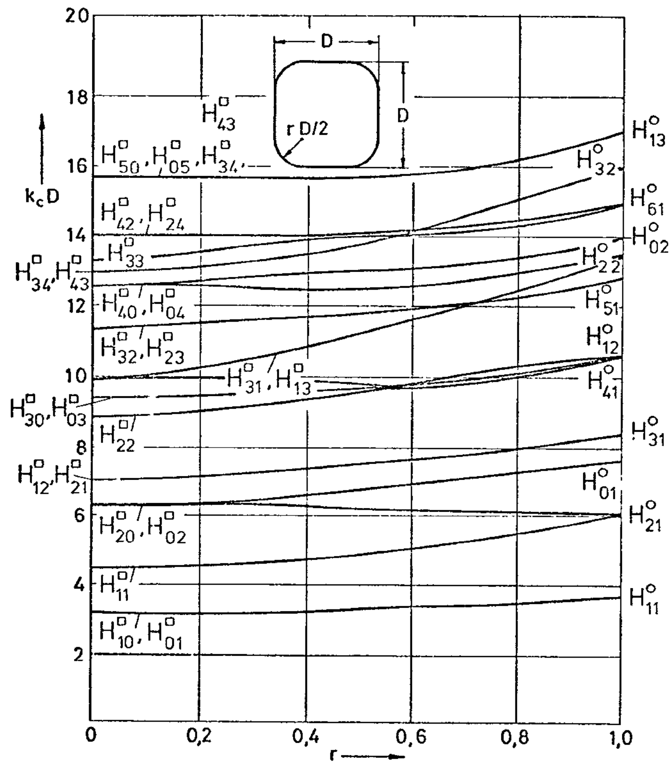

FIG.2 CUTOFF WAVE NUMBER k_c OF H-MODES IN SQUARE TO ROUND WAVEGUIDE TRANSITIONS WITH CORNERS ROUNDED OFF

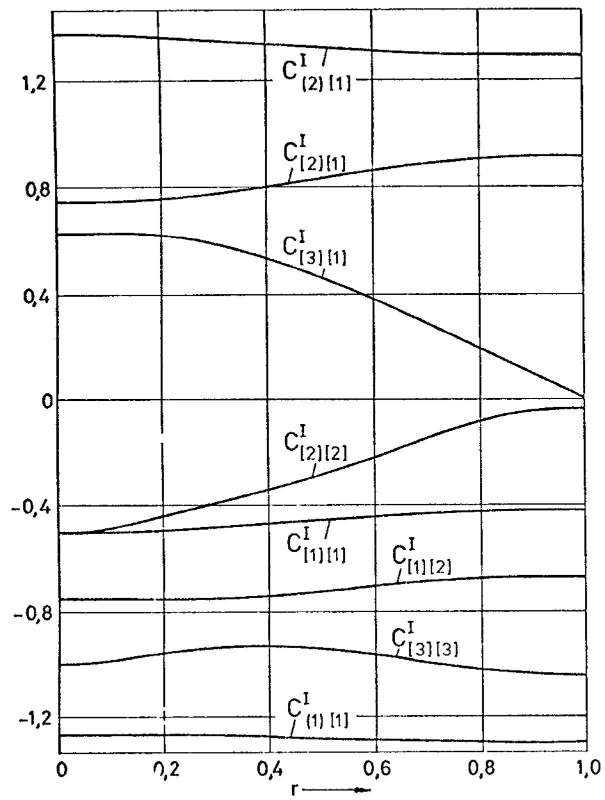

Fig 3

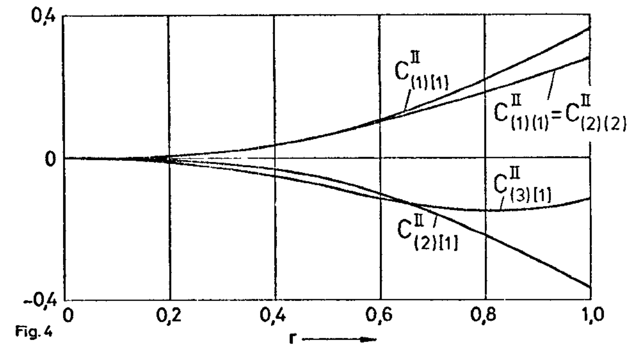

Fig. 4

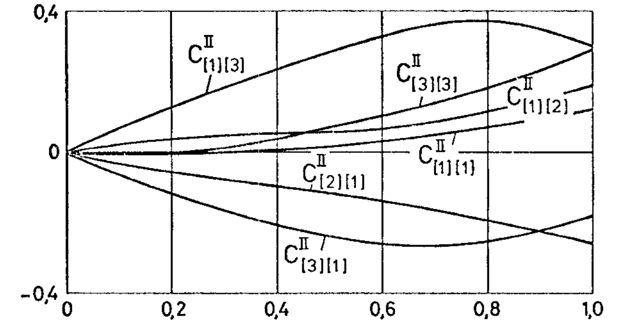

Fig 5

FIG. 3, 4, 5 COUPLING FACTORS FOR RECTANGULAR TO ROUND WAVEGUIDE TRANSITIONS

MODE DESIGNATION:

RECTANGULAR ROUND	INDEX	RECTANGULAR ROUND	INDEX
$(E^{\square}_{12} - E^{\circ}_{11})$-MODE	(1)	$(H^{\square}_{10} - H^{\circ}_{11})$-MODE	[1]
$(E^{\square}_{14} - E^{\circ}_{12})$-MODE	(2)	$(H^{\square}_{30} - H^{\circ}_{12})$-MODE	[2]
$(E^{\square}_{32} - E^{\circ}_{31})$-MODE	(3)	$(H^{\square}_{12} - H^{\circ}_{31})$-MODE	[3]
$(E^{\square}_{34} - E^{\circ}_{51})$-MODE	(4)		

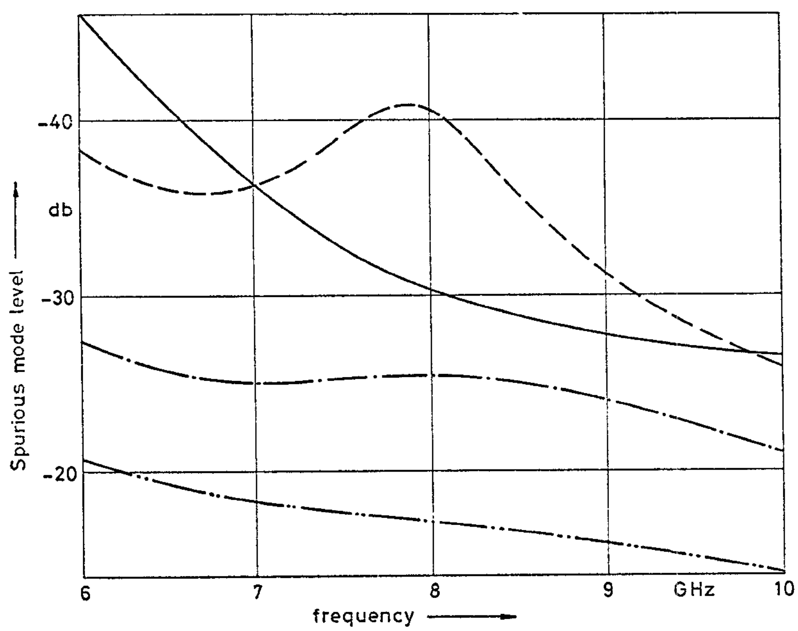

FIG. 6 SPURIOUS $(E^{\square}_{12} - E^{\circ}_{11})$-MODE GENERATION IN A DOMINANT $(H^{\square}_{10} - H^{\circ}_{11})$-MODE TRANSITION FROM SQUARE TO ROUND WAVEGUIDE WITH DIFFERENT TAPER FORMS

——————— TAPER ACCORDING TO EQU. (1)
- - - - - - - - - (2)
— · — · — TAPER ACCORDING TO EQU. (3)
— · · — · · (4)